CS-111: Written Assignment 1

Submission instructions:

Submit your answers to the following questions in a single pdf file on Canvas & Gradescope. Your work is due by **11:59 p.m. on Wednesday, the 1st of May**.

Questions:

1) Consider a signal blurring system. Every sample of the output signal is generated by averaging the values of the sample itself, and its left and right neighbors in the input signal. (Assume that the samples at the boundary of the input signal are zero). **[4+2+3=9]**

   a. Is this system linear? Prove your answer.
   b. What is the impulse response of the system?
   c. How would this impulse response change if a larger neighborhood of five sample is considered?

2) In gradient-based edge detection algorithms, a gradient is approximated by a difference. Three such difference operations are shown below. This difference can be viewed as a convolution of $f(x,y)$ with some impulse response of a filter $h(x,y)$. Determine $h(x,y)$ for each of the following difference operators. **[2+2+3=7]**

   a. $f(x,y) - f(x-1,y)$
   b. $f(x+1,y) - f(x,y)$
   c. $f(x+1,y+1) - f(x-1,y+1) + 2[f(x+1,y) - f(x-1,y)] + f(x+1,y-1) - f(x-1,y-1)$

3) System A is an "all pass" system, i.e. its output is identical to its input. System B is a low-pass filter that passes all frequencies below the cutoff frequency without change and blocks all frequencies above. Call the impulse response of system B, $b[n]$. **[2+2+3+3+2+3=15]**

   a. What is the impulse response of system A?
   b. How would the impulse response of system B need to be changed to make the system have an inverted output (i.e., the same output, just changed in sign)?
   c. If the two systems are arranged in parallel with added outputs, what is the impulse response of the combination?
   d. If the two systems are arranged in parallel, with the output of system B subtracted from the output of system A, what is the impulse response of the combination?
   e. What kind of filter is the system in (d)?
   f. In this problem, system B has the ideal characteristic of passing certain frequencies "without change." How would the outputs of the systems described in (c) and (d) be affected if the low-pass filter delayed (i.e., shifted) the output signal by a small amount, relative to the input signal?

4) You can blur an image using a filter B and find the horizontal gradient of an image using the filter G, where B and G are given as follows. You are also assured that both of these are linear methods. **[4+3+3+3=13]**

$B =$

1/4	1/4
1/4	1/4

$G =$

-1	+1
-1	+1

a. Prove that the method of finding the gradient is linear. (You have already proved the blurring is linear in Question 1).
   Hint: *This has nothing to do with the filters. Think about the functions that the filters are achieving. You can give the proof in 1D.*
b. You are asked to apply B to the image to generate a blurred image and then apply G to the result to find the gradient of the blurred image applying G. How would the output change if you apply G first and then B?
c. If you are asked to apply one single filter S, instead of two filters in succession, how would you find S?
d. If you are asked to design a single filter S that would achieve the effect of adding the blurred image and the gradient image created by applying B and G separately, how would you find S?

5) You would like to detect edges in an image. You can use a curvature-based method C or a gradient-based method G. **[4+4+4+4=16]**

   a. Would using C require using a single or multiple convolution operations? Justify your answer.
   b. Would using G require using a single or multiple convolution operations? Justify your answer.
   c. Edge detector filters usually combine a low pass filter with a curvature or gradient filter. Why?
   d. How does the width of this low pass filter affect the resolution of the edges you would detect?

6) Consider a binary image created by an edge detection method that marks all the edge pixels. I would like to use Hough transform to see if this image has any circles. **[1+3+3=7]**

   a. What is the dimension of the Hough space?
   b. Write the equation of the corresponding Hough space entity for each pixel (x, y)?
   c. Infer from this equation the shape in the Hough space that corresponds to each pixel (x, y)?

7) Consider a one dimensional signal x of length 16 where sample i is given by $x[i] = 2sin(\pi i/4) + 3cos(\pi i/2) + 4cos(\pi i) + 5$. **[2+4+4=10]**

   a. What is the length of each of the arrays Xc and Xs?
   b. Write out the array Xc and Xs?
   c. Convert the Xc and Xs representation to that of magnitude M and phase P. Write out the arrays M and P.

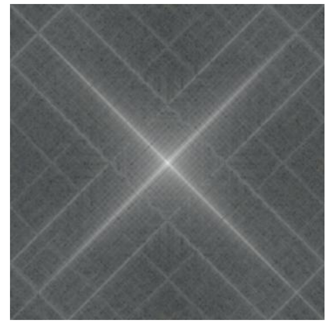
8) Suppose that you have a gray image corrupted by salt-and-pepper noise, as shown in the matrix below **[3+3+4 = 10]**

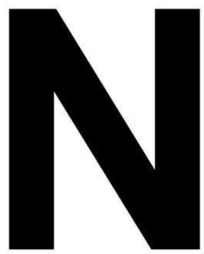
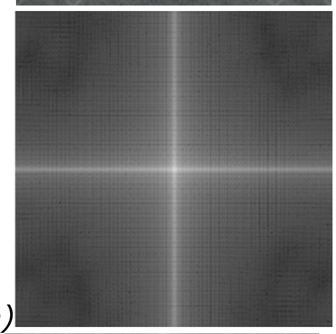
0.5	0.5	0.5	0.5	0.5
0.5	0.0	0.5	0.5	0.5
0.5	0.5	0.5	0.5	0.5
0.5	0.5	0.5	1.0	0.5
0.5	0.5	0.5	0.5	0.5

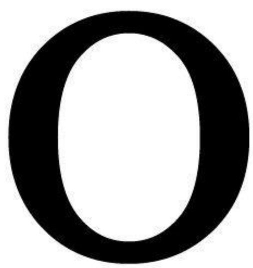
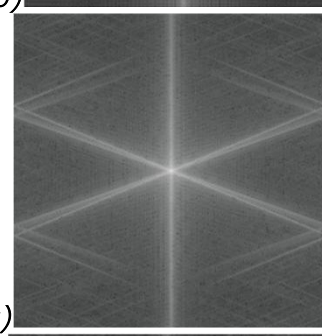
Note: You don’t need to apply the filter on the border pixels.

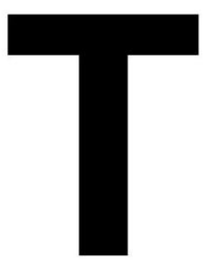
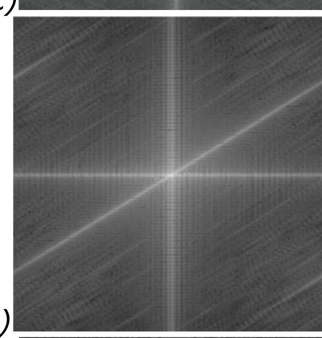
a. What is the filtered result after applying 3×3 box filter? Draw the matrix
b. What is the filtered result after applying 3×3 median filter? Draw the matrix
c. Compare the results you got from (a) and (b), explain why median filter is better in filtering out salt-and-pepper noise.

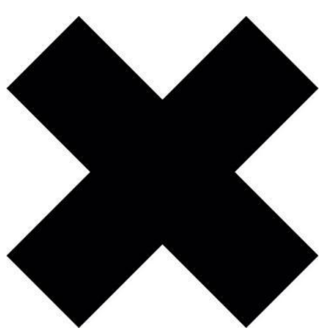
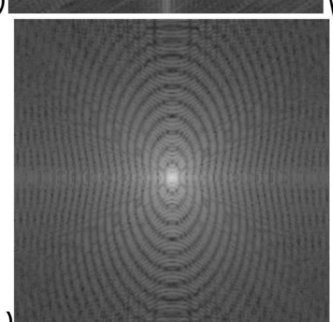
9) Match the images from left column with the image of the magnitude of their DFT from right column. **[5]**

i)  *a)* 

ii)  *b)* 

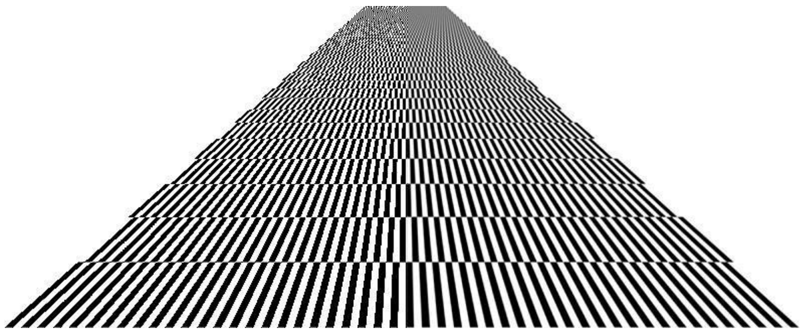
iii)  *c)* 

iv)  *d)*  \

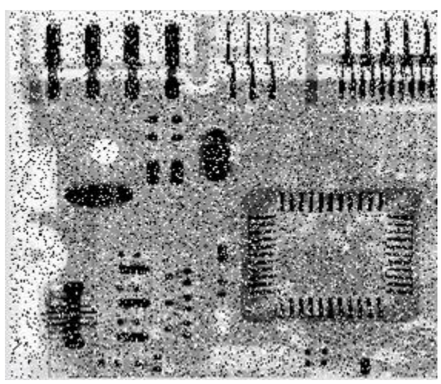
v)  *e)* 

10) What is the corresponding artifact for each of the image below and provide one method to reduce it. **[8]**

Hint: *common artifacts include **aliasing**, **noise** (salt-and-pepper noise, random noise, periodic noise), **quantization error** etc.*

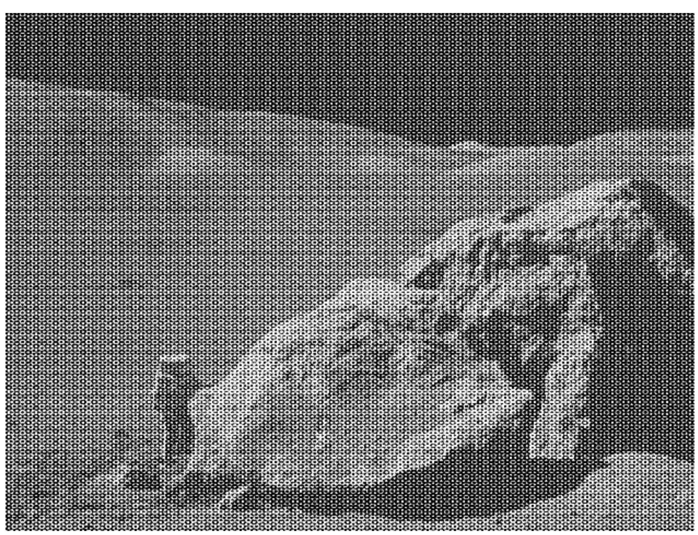


a)



b)

c)



d)